MATHEMATICAL LOGIC HOMEWORK 4

Due Friday, March 8.

Problem 1. *Exercise 5.28 from the lecture notes.*

Problem 2. *Suppose that U is a principal ultrafilter on I i.e. for some $a \in I$, $U = \{A \subset I \mid a \in A\}$. Fix this a.*

- *Suppose that $\{\mathcal{M}_i \mid i \in I\}$ is a family of $\mathcal{L}$-models. Show that the ultraproduct $\prod_{i \in I} \mathcal{M}_i/U$ is isomorphic to $\mathcal{M}_i$.*
- *Suppose that $\mathcal{M}$ is in $\mathcal{L}$-model. Show that the ultrapower $\mathcal{M}^I/U$ is isomorphic to $\mathcal{M}$.*

Problem 3. *Exercise 5.37 from the lecture notes.*

Problem 4. *Let U be a non-principal ultrafilter on the set of prime numbers. For each prime p, let F_p^{alg} be the algebraic closure of F_p, the filed with p elements. Prove that $\prod F_p^{alg}/U$ is an algebraically closed field of characteristic 0.*

Problem 5. *Exercise 5.39 from the lecture notes.*